UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

(4)

AD-A209 635

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AI Memo 1115	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Design Considerations for an Earth Based Flexible Robotic System		5. TYPE OF REPORT & PERIOD COVERED memorandum
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Andrew D. Christian and Warren P. Seering		8. CONTRACT OR GRANT NUMBER(s) N00014-86-K-0685
9. PERFORMING ORGANIZATION NAME AND ADDRESS Artificial Intelligence Laboratory 545 Technology Square Cambridge, MA 02139		10. PROGRAM ELEMENT PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Advanced Research Projects Agency 1400 Wilson Blvd. Arlington, VA 22209		12. REPORT DATE April 1989
		13. NUMBER OF PAGES 16
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) Office of Naval Research Information Systems Arlington, VA 22217		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Distribution is unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

DTIC ELECTE JUN 26 1989 S E D

18. SUPPLEMENTARY NOTES

None

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

robot
flexible
design
vibration

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This paper provides insights into the problems of designing a robot with joint and link flexibility. The relationship between the deflection of the robot under gravity is correlated with the fundamental frequency of vibration. We consider different types of link geometry and evaluate the flexibility potential of different materials. Some general conclusions and guidelines for constructing a flexible robot are given.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
ARTIFICIAL INTELLIGENCE LABORATORY

A.I.Memo No. 1115 May, 1989

Accession For
NTIS GRA&I
DTIC TAB
Unannounced
Justification
By
Distribution/
Availability Codes
Dist Avail and/or Special
A-1

Design Considerations for an Earth Based Flexible Robotic System

Andrew D. Christian
Warren P. Seering

DTIC COPY INSPECTED 6

Abstract: This paper provides insights into the problems of designing a robot with joint and link flexibility. The relationship between the deflection of the robot under gravity is correlated with the fundamental frequency of vibration. We consider different types of link geometry and evaluate the flexibility potential of different materials. Some general conclusions and guidelines for constructing a flexible robot are given.

Acknowledgements: This report describes research done at the Artificial Intelligence Laboratory of the Massachusetts Institute of Technology. This material is based upon work supported under a National Science Foundation Graduate Fellowship. Funding for the work was provided in part by the Office of Naval Research University Research Initiative Program under Office of Naval Research contract N00014-86-K-0685 and in part by the Charles Stark Draper Laboratory, grants #DL-H-285399 and #CSDL-5678.

©1989 Massachusetts Institute of Technology.

Published in the 1989 IEEE International Conference on Robotics and Automation

1 Introduction

For many years people have been interested in the control of flexible structures. Problems caused by flexibility show up frequently, ranging from automated machinery that has to pause momentarily to allow vibration to damp out to instability in non-colocated control loops. Space-based structures and robots are particularly susceptible to severe flexibility problems because they tend to be long, slender and extremely lightweight.

As the accurate control of a flexible robot is a complicated task, researchers have begun by building simplified flexible robots. The simplest form is a single mass-beam system, explored by many researchers including [1,2]. Two degree of freedom, planar models have been built both as systems with stiff links/flexible joints [3], as a single flexible link/fast end effector [4] system, and as two flexible links with stiff joints [5].

Planar, low degree of freedom robots are good for experimenting with control techniques at low frequencies of vibration. However, they cannot reproduce the complications of a true space-based robot: vibrations with coupled bending and torsional modes and joints exciting modes orthogonal to their plane of action. The practical problems of controlling such an arm has lead researchers to building three dimensional, flexible robots, as by [6].

Unfortunately, a flexible earth-based robot suffers from an inseparable problem: gravity. Unless you build in some form of compensation to eliminate the effects of gravity, your robot will sag. The standard way to get around sagging is to make the links of the robot directionally stiff with a higher stiffness vertically than horizontally. But directional stiffness can eliminate some of the very effects that you are studying. Consider a standard two link robot model with three degrees of freedom: two revolute joints at the base and a revolute joint at the shoulder which is the same plane as one of the base joints. Rotation of the base or movement of the two co-planar joints will excite vibration in the radial sense, but only rotation of the base will excite vibration perpendicular to this. The directional stiffness has artificially raised the natural frequency in a direction that may cause the most problems in a true space-based arm.

The motivation for this research was the intent to design and build an anthropomorphic, two link, three degree of freedom test fixture that would exhibit both link and joint flexibility. The design needed to behave like a teleoperated space robot, complete with low frequencies of vibration and coupled modes. This paper deals with the underlying issues that came up while designing links for the robot that could give the desired flexibility and not break. The first issue is how the endpoint deflection of the robot under gravity relates to the lowest natural frequency of vibration of the robot. The second issue deals with the best shapes and materials to be used for the flexible links of the robot.

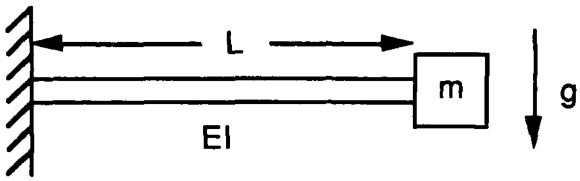

Figure 1: Single cantilever beam with a mass

2 Vibration Considerations

When building a robot that will have primary modes of vibration in all directions, a natural design issue is what will be the lowest natural frequency of the robot. A low fundamental frequency has several advantages and one clear disadvantage. The low fundamental frequency is easy to observe and record. It allows the higher modes of vibration to occur at frequencies that may also be visible. But the disadvantage is that a flexible arm will sag under gravity.

It turns out that the deflection of the arm under gravity is a very good way to estimate its natural frequency of vibration, and vice-versa. To demonstrate this, we will begin by deriving the relationship between the endpoint deflection of a single beam and its natural frequency. Then we demonstrate that for real two link flexible systems, the formula relating endpoint deflection to natural frequency forms a useful estimate of the system's natural frequency.

2.1 Single Beam Under Gravity Loading

There is a useful relation between the natural frequency of vibration and the deflection of a single beam under gravitational loading, mentioned by [7]. If we consider a single cantilever beam with a mass (as shown in Figure 1) and assume that it behaves as a Bernoulli-Euler beam we have

$$\delta_L = \frac{mgL^3}{3EI} \tag{1}$$

where δ_L is the deflection of the end of the beam and the mass of the beam is considered to be negligible. The spring constant of the beam that relates the endpoint deflection to the force acting at the end can be written as

$$K = \frac{mg}{\delta_L} \tag{2}$$

To a good approximation, the lowest natural frequency of the beam is given by

$$f_g = \frac{1}{2\pi}\sqrt{\frac{K}{m}} \tag{3}$$

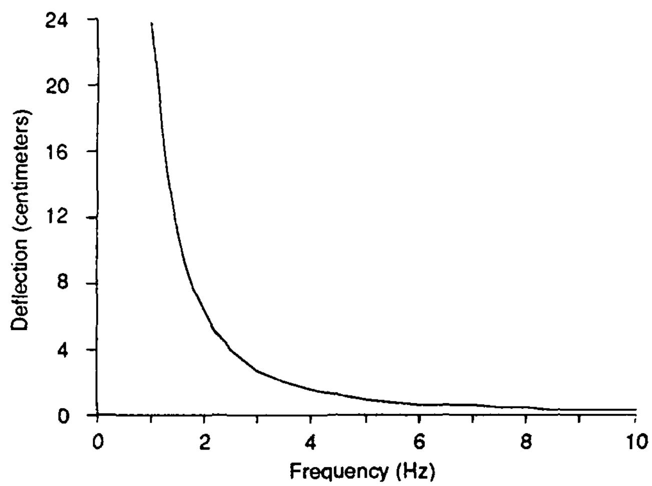

Figure 2: Deflection of a Single Beam under Gravity

where f_g is in Hertz. By combining equations (2) and (3) we express the frequency of vibration of the cantilevered beam as

$$f_g = \frac{1}{2\pi}\sqrt{\frac{g}{\delta_L}} \tag{4}$$

Hence the natural frequency of vibration of the beam can be approximated as a function of its deflection under gravitational loading. We can also write the equation this way:

$$\delta_L = \frac{g}{4\pi^2 f_g^2} \tag{5}$$

or using $g = 9.8$ m/sec^2, we have

$$\delta_L = \frac{25}{f_g^2} \text{ centimeters} \tag{6}$$

$$f_g = \frac{5}{\sqrt{\delta_L}} \text{ Hz} \tag{7}$$

where δ_L is in centimeters and f_g is in Hertz. This is displayed graphically in figure 2. Note particularly that frequencies under a few hertz result in extremely large deflections under gravity.

2.2 Two Beams Under Gravity Loading

Equations (6) and (7) are useful formulas to keep in mind when you are considering the behavior of a single link flexible robot. Now consider a two link robot modeled as a two beam system with a joint mass m_1 between the links and an additional payload

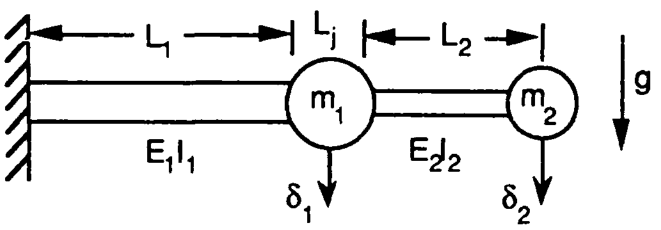

Figure 3: Two Cantilevered Beams with Masses

mass m_2 suspended at the ends, as shown in Figure 3. To make this a realistic model of a robot arm, we include the length of the first mass as L_j. We simplify the analysis with two assumptions. We assume that the masses of the beams are negligible as compared to the masses of the joint and the payload. We assume that the beams behave as Bernoulli-Euler beams in bending. The flexibility matrix of the system (see [8]) is

$$\begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \tag{8}$$

where δ_1, δ_2 are the deflections at m_1 and m_2 from the forces F_1 and F_2. a_{ij} refers to the deflection at mass i due to a unit force at mass j. The values of a_{ij} can be found as

$$a_{11} = \frac{1}{K_1}[1 + \frac{3L_j}{2L_1} + \frac{3{L_j}^2}{4{L_1}^2}] \tag{9}$$

$$a_{12} = a_{21} = \frac{1}{K_1}[1 + \frac{3L_j}{4L_1} + \frac{3}{2{L_1}^2}(L_1 + L_j)(L_2 + L_j)] \tag{10}$$

$$a_{22} = \frac{1}{K_2} + \frac{1}{K_1}[1 + \frac{3(L_2 + L_j)}{L_1} + \frac{3}{{L_1}^2}(L_2 + L_j)^2] \tag{11}$$

where we have substituted $K_1 = 3E_2I_1/{L_1}^3$ and $K_2 = 3E_2I_2/{L_2}^3$. Using the standard assumption of harmonic motion and replacing the forces F_1 and F_2 by inertia forces $F_i = -m_i\ddot{\delta}_i = \omega^2 m_i \delta_i$ we then find the vibrational frequencies by calculating the determinant and solving for ω from

$$\det \begin{vmatrix} (a_{11}m_1 - \frac{1}{\omega_n^2}) & a_{12}m_2 \\ a_{21}m_1 & (a_{22}m_2 - \frac{1}{\omega_n^2}) \end{vmatrix} = 0 \tag{12}$$

which can be solved explicitly for $f_n = \omega/2\pi$ as

$$f_n = \frac{\sqrt{2}}{2\pi}\Big(a_{11}m_1 + a_{22}m_2 + ((a_{11}m_1 + a_{22}m_2)^2 - 4(a_{11}a_{22}m_1m_2 - {a_{12}}^2 m_1 m_2))^{1/2}\Big)^{-1/2} \tag{13}$$

The endpoint deflection of this system due to gravitational loading is

$$\delta_{tip} = a_{21}m_1g + a_{22}m_2g \tag{14}$$

If we substitute this into (4), we get the approximation

$$f_g = \frac{1}{2\pi}\sqrt{\frac{1}{a_{21}m_1 + a_{22}m_2}} \tag{15}$$

This value can be compared to the expected vibrational frequency from equation (13).

In fact, it is easy prove that $f_g \leq f_n$. So, assuming that the vibrational analysis is reasonably good, then the quick f_g calculation will be a lower bound for the actual lowest frequency of vibration of the system. As long as f_n is not too much greater than f_g, f_g forms a useful estimate of the natural frequency. In the next section of the paper, we will empirically demonstrate that f_g is a good estimate for real two beam, two mass systems.

Equations (13) and (15) depend fundamentally on the assumption that the system can be considered to be a long beam with a mass on the end. In fact, when $m_2 \to \infty$ or $K_1 \to \infty$, $f_g \simeq f_n$. So as the system more closely resembles either a single mass/beam system (where m_1 is completely negligible) or a system with just an end mass (where K_1 is so stiff that you can treat K_2 as built into a wall), the closer it matches the ideal case. If you keep reasonably "balanced" values for your parameters, in the sense that each beam participates in the vibration and neither mass strongly exceeds the other, $f_n \simeq f_g$ is a good approximation.

2.3 Comparison of Vibrational Formulas

We would like to compare the values of f_g and f_n to determine the usefulness of the "tip deflection under gravity" approximation. The vibrational frequency f_n is a function $f_n = f_n(L_1, L_2, L_j, m_1, m_2, k_1, k_2)$ which is too complex to graph easily. Instead, we pick two sets of parameters and see how varying them affects the ratio of estimated vibrational frequency to actual vibrational frequency.

Two typical cases of beam configurations are displayed in Table 1, one aluminum and one steel. These values were chosen as representative of the types and sizes of systems that we have considered in the course of our research. Each of these robot configurations has a natural frequency of vibration of approximately 3.5 Hertz. Additionally, the K values were choosen so that each beam participates equally in the vibration; that is, if you assume a round cross section of beam, the maximum stress level experienced in beam L_1 under gravity loading is the same as that experienced in L_2.

As shown in Figures 4 and 5, varying the length of the first and second links does not change the ratio by more than a few percent. If we then hold the lengths constant and change the weight of the first mass and the stiffness of the first link, as shown in

Link	L_1 (cm)	L_2 (cm)	L_j (cm)	m_1 (kg)	K_1 (N/cm)	m_2 (kg)	K_2 (N/cm)
Steel	45	45	20	7	220	1.4	12
Aluminum	60	45	10	4.5	210	4.5	80

Table 1: Parameter Values for Beam Comparisons

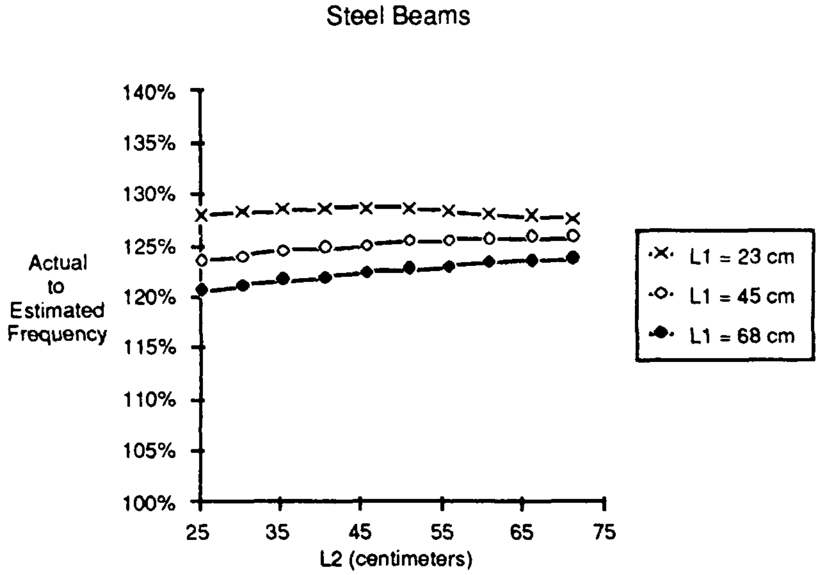

Figure 4: Variation in the ratio of frequency estimate to actual natural frequency as a function of link length

Figures 6 and 7, the change is much larger but still within 10% of the original ratio. Changing the first mass has the largest effect on the estimate but even then f_g is an estimate good to within 20% as long as m_1 is kept small.

After trying different values of lengths, masses, and stiffnesses, it becomes evident that the estimate of vibrational frequency based on static gravitational deflection is very good. To a reasonable approximation, it is safe to say that the vibrational frequency estimate based on endpoint deflection under gravity loading is good to about 30% of the actual value. As f_g forms a lower bound to the vibrational frequency, if you want a system with the lowest vibrational frequency for a given endpoint deflection, you can't do any better than $5/\sqrt{\delta_L}$ (see equation 7). This result is useful because it is often easier to calculate an endpoint deflection for a complex system than a resonant frequency.

It is important to realize that the gravitational deflection versus natural frequency

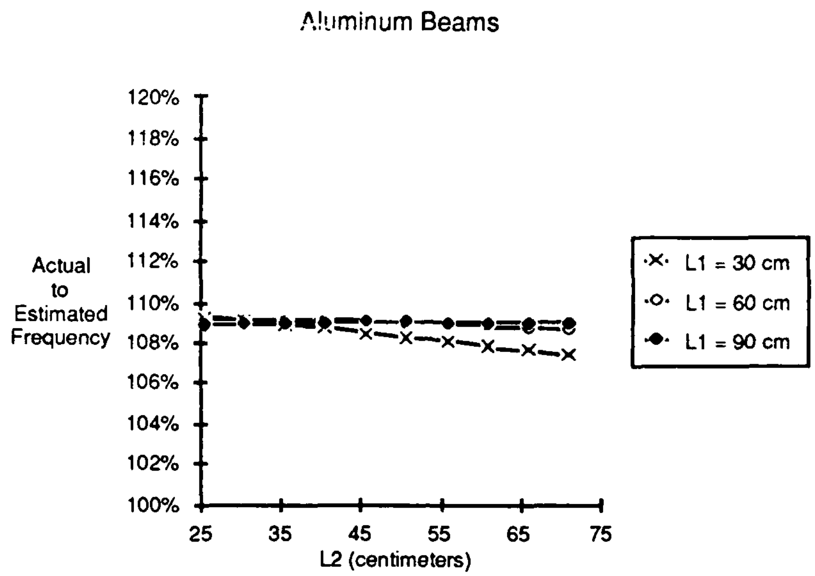

Figure 5: Variation in the ratio of frequency estimate to actual natural frequency as a function of link length

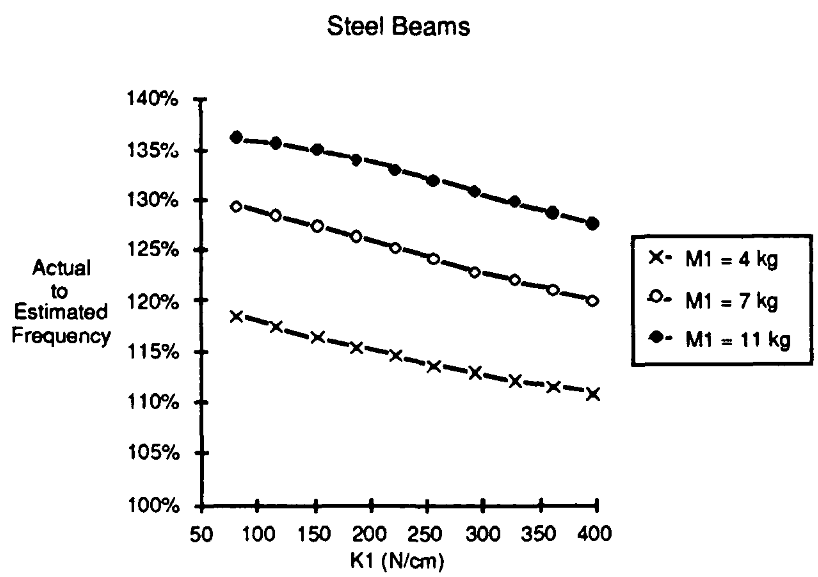

Figure 6: Variation in the ratio of frequency estimate to actual natural frequency as a function of first link stiffness and joint mass

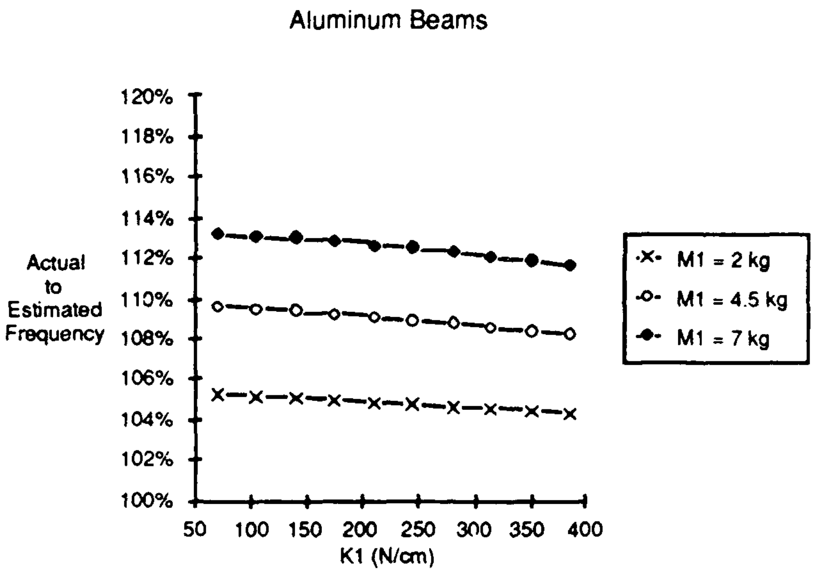

Figure 7: Variation in the ratio of frequency estimate to actual natural frequency as a function of first link stiffness and joint mass

relation does not depend on the length of the beams (see equation 6). If you want a system that vibrates at 1 Hertz, you must live with a endpoint deflection of at least 25 centimeters under gravity, regardless of how long your beams are. Fortunately, the required endpoint deflection falls off quickly as the vibrational frequency is increased. (see Figure 2) The robot we are building has a target frequency of 3Hz, which should give it a little over a one inch endpoint deflection due to gravity.

Finally, remember that this relationship has been derived based on the endpoint deflection under a gravitational load. Horizontal estimates are found just as easily if you assume that the robot has been placed on its side and calculate how far the endpoint deflects.

2.4 Length of the Arms

If you are designing a robot that will have its fundamental frequency of vibration at one Hertz, the robot is going to sag under its own weight at by least 25 cm. A one meter arm will probably not work. But if you want to have the end of the robot vibrating with an amplitude of 5 centimeters, how flexible should you make the links? We can calculate the level of stress inside of a link for a given endpoint deflection.

In terms of a single mass system with a constant cross-section beam with a given

tip deflection δ_{tip}, we write the equation

$$\sigma = \frac{My}{I} \tag{16}$$

where M is the bending moment in the bar, y is the maximum distance from the neutral axis and I is the moment of inertia. The bending moment is

$$M = FL = K\delta_{tip}L = \frac{3EI}{L^2}\delta_{tip} \tag{17}$$

where F is the force required to produce an end deflection δ_{tip} and L is the length of the beam. Combining equations (16) and (17), we have

$$\sigma_{max} = \frac{3Ey}{L^2}\delta_{tip} \tag{18}$$

Equation (18) shows that the stress level in the link is directly proportional to the tip deflection. To keep the bending stresses to a minimum for a given tip deflection and natural frequency, you should make the link as long as possible.

3 Link Types

The first step in designing a flexible robot normally includes deciding how long the links will be, how much torque will be available at the joints, and what the approximate weights and sizes of the joints will be. Once these numbers have been selected, you need to pick the actual shape of the links and material used in the links. There is a tradeoff here between the three key parameters: stiffness, mass, and stress level. Typically the stiffness will have been set by the decision of what the lowest frequency of vibration will be. Then you must design links for this stiffness while keeping the stress level and mass of the link to a minimum.

3.1 Springs

In the case of the robot we are currently building, it worked out that in the initial design that a 3 Hertz vibration mode put uncomfortably high stresses on the links. At the time we were calculating based on links made out of solid, straight bars of aluminum. One option that we looked at to eliminate this undesirable stress level was to replace the aluminum bar with a coil spring. At first glance, the coil spring seemed to be the perfect flexible link; not very stiff and able to undergo large deformations without yielding.

It turns out that springs are *too* flexible for their weight. We can compare the weight and flexibility of a straight link made out of a bar of metal with diameter d to a spring made out of the same bar with wire diameter d, but coiled into a helix. Consider the overall lengths of the two links to be the same.

First, we compare the bending stiffness of the spring to the bending stiffness of the bar. The general equation of bending for a spring is

$$\kappa = \frac{\tau}{\theta} = \frac{Ed^4 p}{32LD}\left(\frac{1}{1+E/2G}\right) \tag{19}$$

where τ is the bending moment, θ is the angle of deformation, p is the pitch of the spring, L is the length, D is the pitch diameter of the spring and E and G are the moduli of elasticity and rigidity. If we assume small angles of deformation, so that $\theta = dy/dx$ and we assume that the spring is fixed at one end and is loaded at the other end by a force F perpendicular to the length of the spring, then we write the deflection of the end of the spring as δ, where

$$\delta = \int_0^L \frac{\tau}{\kappa} = \frac{F}{\kappa}\int_0^L (L-x)\,dx \tag{20}$$

or

$$\delta = \frac{FL^2}{2\kappa} \tag{21}$$

Note that δ is not the axial deflection of the spring but a measure of how far the spring bends.

Now we find the effective spring constant of the spring as

$$K_s = \frac{F}{\delta} = \frac{Ed^4 p}{32L^3 D} \tag{22}$$

where we have made the approximation $E = 2G$. For the round bar with the same diameter d, we write

$$K_{bar} = \frac{3EI}{L^3} = \frac{3\pi Ed^4}{64L^3} \tag{23}$$

Combinining equations (22) and (23), we find that

$$\frac{K_s}{K_{bar}} = \frac{2p}{3\pi D} \tag{24}$$

For a practical spring, it is safe to assume that $p < D$, so K_s is at most 20% of K_{bar}.

The weight of the spring can be compared to the weight of the straight bar . The weight of the spring is given by

$$W_s = \rho\pi d^2 L\sqrt{1+\left(\frac{\pi D}{p}\right)^2} \tag{25}$$

and the weight of the straight bar is

$$W_{bar} = \rho\pi d^2 L \tag{26}$$

Then, if we assume that $(\pi D/p)^2 >> 1$, we get

$$\frac{W_s}{W_{bar}} = \frac{\pi D}{p} \tag{27}$$

Again, it is safe to assume that $p < D$, so the weight of a spring made of a coil of wire is at least 3 times the weight of a link made of a straight piece of that wire.

We conclude that the for a given cross-section d, the spring weighs at least three times as much as the straight bar and has at best 20% of the bending stiffness. If you want a very low bending stiffness and don't care about weight, the spring is the way to go. But if you want a given stiffness, then the spring is going to be at least 15 times as heavy as a straight bar of metal. Even though this allows you to not worry about breaking your link, the additional weight penalty in most systems is prohibitive.

3.2 Material Choice

For a given bending stiffness, the material used for the link will determine its size, weight, and how much deflection it will undergo before yielding. A material with a high yield stress is not necessarily the best choice. If it has a high modulus of elasticity, *the link will have to be thinner* to get the same bending stiffness. The smaller moment of inertia will result in a higher stress level than that of a link made of a material with a high yield strength and a low modulus of elasticity. A similar situation exists with weight.

To compare materials, we first assume that the link is a solid bar of metal with a round cross section of diameter d. We choose a round cross section for two reasons: First, it has the same stiffness when bent in any direction, where the stiffness is deflection of the end of the link with respect to the force applied at the end. Second, for a given bending moment and stiffness K, a solid bar experiences a lower bending stress than a hollow bar. (Consider two bars with the same moment of inertia; one solid and one hollow. The hollow bar will have a larger diameter and the bending stress is directly proportional to the diameter.) If we assume that we know the length of the link, the stiffness, and the applied bending moment, we can calculate the stress and weight of the link as a function of density, modulus of elasticity and yield strength.

Start with the standard formulas

$$\sigma = \frac{Md}{2I} \tag{28}$$

and

$$K = \frac{3EI}{L^3} \tag{29}$$

where E is the modulus of elasticity of the metal, M is the applied bending moment, I is the moment of inertia of a round cross section and L is the length of the beam. Rewrite equation (29) as

$$I = \frac{L^3 K}{3E} = \frac{\pi}{64} d^4 \tag{30}$$

or

$$d = \sqrt[4]{\frac{64 L^3 K}{3\pi E}} \tag{31}$$

To find the stress in the beam, combine equations (28) and (31)

$$\sigma = \frac{6M}{KL^2} \sqrt[4]{\frac{KE^3}{12\pi L}} \tag{32}$$

Equation (32) shows that with L, K and M fixed for a beam, the material with the lowest fraction of stress to yield stress will be the one with the largest value of ψ, where ψ is defined as:

$$\psi = \frac{\sigma_{yield}}{\sqrt[4]{E^3}} \tag{33}$$

The larger the value of ψ, the larger the bending moment the link will withstand before yielding.

Another material comparison is the weight of the bar as a function of the length and the spring constant. Using equation (31) and taking ρ as the density of the link, we get

$$W = \frac{\pi \rho d^2 L}{4} = 4\rho \sqrt{\frac{\pi L^5 K}{3E}} \tag{34}$$

where W is the weight of the link. For a given L and K, the beam with the lowest weight will be the one that minimizes ϕ, where ϕ is given by:

$$\phi = \frac{\rho}{\sqrt{E}} \tag{35}$$

The larger the value of ϕ, the heavier link.

The optimal link material would have a large ψ and a small ϕ. In practice, no material is optimal although Titanium comes close. Table 2 shows the value of ψ and ϕ worked out for a number of different common metals. Titanium has the best strength to stiffness and 2014-T6 Aluminum is a practical, lightweight alternative although its resistance to fatigue is limited.

4 Joint Flexibility

Robots have two types of flexibility; in the joints and in the links. Joint flexibility appears to the system as springs in series with the links. To illustrate this, look at

Material	σ_y (MPa)	E (GPa)	ρ (kg/m^3)	ψ	ϕ
1100-0 Aluminum	34	72	2800	1.4	330
2014-T6 Aluminum	415	72	2800	16.8	330
6061-T6 Aluminum	275	72	2800	11.1	330
1015 Steel	324	207	7700	5.9	535
4140 Steel	655	207	7700	12.0	535
Magnesium Alloy	240	45	1800	13.8	268
Titanium Alloy	830	114	4400	23.4	412

Table 2: Link Material Comparison

the two link model shown in Figure 3. Now add in a torsional spring of value κ_1 where the first link meets the wall and a torsional spring of value κ_2 between the links. Equations (9), (10), and (11) relate the deflection of a point to a unit force applied at a point, so we can modify them to take into account the torsional springs:

$$a'_{11} = a_{11} + \frac{L_1 + L_j/2}{\kappa_1} \tag{36}$$

$$a'_{12} = a'_{21} = a_{21} + \frac{L_1 + L_j/2}{\kappa_1} \tag{37}$$

$$a'_{22} = a_{22} + \frac{L_1 + L_2 + L_j}{\kappa_1} + \frac{L_2 + L_j/2}{\kappa_2} \tag{38}$$

These modified flexibility values can be substituted in equations (13) and (15) to give the estimates for the lowest natural frequency of the system. We still have f_g as a lower bound for f_n, and assuming that the springs are not extremely flexible, f_g still gives a good approximation of the lowest natural frequency.

5 Conclusion

The endpoint deflection of a robot in a gravity field due to bending in the links and flexiblity in the joints forms a lower bound for the natural frequency of vibration of the system. When all of the links are flexible and the weight of the payload is not negligible in comparison to the weight of the joints, this estimate is very close to the true frequency. As this estimate forms a lower bound, if your goal is to minimize the fundamental natural frequency of the system, you should make links that are as long as possible and have small masses at the joints. The "tip deflection under gravity" calculation is not restricted to the vertical case; we can put an imaginary

gravitational field pointing in any direction, calculate the deflection of the robot, and estimate the natural frequency of vibration.

For maximum flexibility with minimum stiffness, the best material choice for the links is titanium. Some aluminum alloys may be inexpensive alternatives, but the fatigue characteristics of aluminum are unacceptable in most applications. Coiled springs give very little stiffness in comparison to their strength, but their weight makes them unusable in all but planar applications.

There are physical difficulties to building a test fixture with a very low frequency of vibration. To keep stress levels low in a flexible robot, long links are prefered, but may not be possible if the robot must fit inside of a laboratory. The longer links have a larger moment of inertia and need larger motors to drive them around. Joints that contain motors carry a great deal of weight in the form of motors, gears, brakes, encoders, and ball bearings. Trying to optimize the robot to have low joint masses, long links, and adequate torque at the joints is difficult. With these constraints in mind, we are currently assembling a three degree of freedom flexible robot. It has a lowest natural frequency of vibration of under 3 Hertz, a payload capacity of 3 pounds and a total reach of at least 51 inches. We expect that it will be operational by June of 1989.

References

[1] **Cannon, Robert H. and Schmitz, Eric,**
"Initial Experiments on the End-Point Control of a Flexible One-Link Robot", *The International Journal of Robotics Research*, Vol. 3, No. 3., Fall 1984.

[2] **Alberts, T.E., Hastings, G.G., Book, W.J., and Dickerson, S.L.,**
"Experiments in Optimal Control of a Flexible Arm with Passive Damping", *Fifth VPISSU/AIAA Symposium on the Dynamics and Control of Large Structures*, 1985.

[3] **Hollars, Michael G. and Cannon, Robert H.,**
"Initial Experiments on the End-Point Control of a Two-Link Manipulator with Flexible Tendons", *ASME Winter Annual Meeting*, November 19, 1985.

[4] **Tilley, S.W., Cannon, R.H., and Kraft, Ray,**
"End Point Force Control of a Very Flexible Manipulator with a Fast End Effector", *ASME Winter Annual Meeting*, December, 1986.

[5] **Bayo, E.,**
"Computed Torque for the Position Control of Open-Chain Flexible Robots", *IEEE International Conference on Robotics and Automation*, April 25-29, 1988.

[6] **Pfeiffer, F. and Gebler, B.,**
"A Multistage-Approach to the Dynamics and Control of Elastic Robots", *IEEE International Conference on Robotics and Automation*, April 25-29, 1988.

[7] **Blevins, Robert D.,**
Formulas for Natural Frequency and Mode Shape, Van Nostrand Reinhold Company, Appendix A, 1979.

[8] **Thomson, William T.,**
Theory of Vibration with Applications, Prentice-Hall, Inc., 1981.

DISTRIBUTION:

Defense Technical Information Center

Computer Sciences Division
ONR, Code 1133

Navy Center for Applied Research in Artificial Intelligence
Naval Research Laboratory, Code 5510

Dr. A.L. Slafkosky
Headquarters, U.S. Marine Corps (RD-1)

Psychological Sciences Division
ONR, Code 1142PT

Applied Research & Technology
ONR, Code 12

Dept. of the Navy
Naval Sea Systems Command
NAVSEA 90

Dr. Charles Schoman
David Taylor Naval Ship R&D Center
NSRDC 18